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Rest Frame Properties of the Proton

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The proton is modeled as three quarks of small current quark mass. The three-body Dirac equation is solved with spin-independent central diagonal linear confining potentials with an attractive Coulombic term in a relativistic three-quark model. Hyperspherical coordinates are used, and the bound state is found analytically. After integrating over the hyperangles, the Hamiltonian is an 8 by 8 matrix of coupled first-order differential equations in one variable, the hyperradius. These are analytically solved in hypercentral approximation. For the $(1/2^+)^3$ ground-state configuration in the nonrelativistic large-quark-mass limit, there are no nodes in the wave function. However, in the extreme relativistic limit of small current quark masses of a few MeV, the expectation value of the number of nodes is about 1.30 when the potential parameters are chosen to reproduce the proton rms charge radius. The quarks are assumed to possess a Pauli anomalous magnetic moment, like that of the electron and muon of $(\alpha/2\pi)(e/m)$. Assuming all three quarks have equal mass, one can fit the rest energy, magnetic moment, rms charge radius, and axial charge of the proton with this relativistic three-body Dirac equation model. The solution found shows the necessity of including all components of the composite three-quark wave function, as the upper component contributes only 0.585 to the norm.

1. INTRODUCTION

The case for nonrelativistic constituent quark masses within the nucleon is that it works (Bowler, 1990; Bhaduri, 1988). The constituent quarks are assumed bound in S states in a potential well coupled to total angular momentum 1/2 for the nucleon. Setting their masses to about 330 MeV allows the proton experimental magnetic moment to be reproduced by the Dirac magnetic moment of the bound quarks. The attractive binding potential is such that

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the total energy of the three-quark system matches the proton rest energy. The potentials are small, and the problem is nonrelativistic due to the large quark masses assumed. Recently, relativistic extensions of the constituent quark model of mesons as a quark–antiquark bound pair have shown the importance of the negative-energy components (Tiemiejer and Tjon, 1993).

This paper is instead concerned with using current quark masses (Particle Data Group, 1994) in a necessarily relativistic three-quark modeling of the nucleon. Current quark masses for the up, down quarks in the nucleon are in the 2- to 10-MeV range. The quark dynamics is assumed to be described by the three-body Dirac equation solved in hypercentral approximation. This properly handles the center-of-mass problem in the rest frame of the system. The proton properties will be calculated using the composite three-quark wave function found from solving the three-body Dirac equation. The masses of the u and d quark will generally be taken as the same, but in the next section, temporarily all three masses will be assumed different.

Central diagonal linear confinement potentials (Semay and Ceuleneer, 1993) between the quarks will be used with an attractive Coulombic term. The attractive Coulombic term is included as reminiscent of one-gluon exchange between quarks. Previously quadratic confining potentials have been used that led to Gaussian-type wave functions (Semay and Ceuleneer, 1993; Strobel and Hughes, 1987; Strobel and Pfenninger, 1987; Strobel and Shitikova, 1996). Here hypercentral potentials with a given linear confinment plus Coulombic dependence in the hyperradius r are considered. These result from hyperangular averages (Strobel, 1996a) of quark–quark potentials that have the same analytic dependence in the interquark separations. These potentials have previously been used in studies of the Ioffe constants of the proton and Roper resonance (Strobel and Shitikova, 1997). The model has three parameters—the quark mass, the confining linear size parameter L, and the attractive Coulombic potential coefficient. These potential coefficients are determined by reproducing the proton rest energy and rms charge radius.

The hyperspherical method has been applied to the three-body Dirac equation (Strobel and Hughes, 1987; Strobel and Pfenninger, 1987; Strobel, 1986), where hyperangular averages of a diagonal central potential and the relativistic kinetic energy operator were evaluated. The basic idea is to use the chain rule of calculus to change the partial derivatives of the kinetic energy operator with respect to r_1, etc., into partial derivatives with respect to the hyperradius. The hyperspherical formulation expands the three-body bound-state wave function into a set of configurations each of which has a hyperradial and a hyperangular factor.

The composite three-quark wave function is labeled by the J and parity quantum numbers of the upper component of the orbitals occupied by the three quarks. The positive-party $(1/2^+)^3$ configuration is studied here. Other configurations (Strobel, 1996b), such as the $(1/2^+)(1/2^-)$ or the $(1/2^-)^2(1/2^+)$, result in Dirac magnetic moments of less than 1 nuclear magneton, and are discarded on this basis. The $(1/2^-)^3$ configuration has a Dirac magnetic moment of 8/3 NM, but has negative parity. Orbital excited configurations such as $(3/2^+)^2(1/2^+)$ are not considered here as being major contributors to the proton ground-state wave function. The $(1/2^+)^3$ configuration with a linear confining potential and massless quarks has a Dirac magnetic moment of 2.763 NM when maximized by varying the linear potential size parameter (Strobel, 1996a). A Pauli anomalous magnetic moment is ascribed to each quark, as exists for the electron and muon per QED, of $(\alpha/2\pi)(e/m)$. The quarks are assumed to be in the $(1/2^+)^3$ configuration of positive parity, with the spins coupled to the proton spin of 1/2. The proton/neutron magnetic moment ratio of -1.46 is close to the valence-quark-only ratio of -1.50. This near agreement is a success of the valence quark modeling. The proton magnetic moment is here described by the Dirac and anomalous magnetic moments from the three quarks. There is no anomalous quark magnetic moment contribution to the neutron for equal-mass quarks. Other contributions to the proton magnetic moment may be small. Attributing the deviation from valence quark contributions to an $SU(2)$ symmetric pion cloud or antiquark–quark contribution (Hogasson and Myhrer, 1988; Barik, *et al.*, 1990) predicts such a pion contribution to be about 0.16 NM. Looking at QCD symmetries, Leinweber (1996) gets a disconnected sea quark contribution of -0.17 NM and a strange quark contribution of 0.25 NM assuming u and d quark masses are equal. Such possible pion, mesonic, strange, or sea quark contributions to the proton magnetic moment are neglected here. Their inclusion would lower the anomalous magnetic moment contribution from the quarks needed to agree with the experimental values. When the model parameters are adjusted to reproduce the proton rms charge radius and magnetic moment, small current quark masses of the order of 10 MeV or less are required by this model. The Dirac magnetic moment of the bound quarks is the dominant contribution to the total magnetic moment, with the Pauli contribution only about 5–20%.

2. THREE-BODY DIRAC EQUATION

The composite field for a three-body system is given by

$$\Phi(x_1, x_2, x_3) = \Psi(x_1)\,\Psi(x_2)\,\Psi(x_3) \tag{1}$$

where the $\Psi(x)$ are single-particle wave functions and $\mathbf{x}$ is a four-vector describing the particle coordinates. One obtains for the composite field the three-body Dirac equation (Barut and Komy, 1985; Barut and Strobel, 1986):

$$\left\{\begin{matrix}[(\gamma^\mu i\partial_\mu - m_1) \otimes \gamma^\mu\eta_\mu \otimes \gamma^\mu\eta_\mu + \gamma^\mu\eta_\mu \otimes (\gamma^\mu i\partial_\mu - m_2) \otimes \gamma^\mu\eta_\mu \\ + \gamma^\mu\eta_\mu \otimes \gamma^\mu\eta_\mu \otimes (\gamma^\mu i\partial_\mu - m_3) - \sum_{i<j} V_{ij}(d_{ij})]\end{matrix}\right\}$$

$$\times \Phi(x_1, x_2, x_3) = 0 \tag{2}$$

Here η_μ is a timelike four-velocity vector of the system. In the center-of-momentum frame, η_μ is (1, 0, 0, 0). d_{ij} is the transverse difference of the two four-vectors x_i and x_j. Here x_{ij} is defined as $(x_i - x_j)^2$, so that the transverse difference is

$$d_{ij} = \{-(x_i - x_j)^2 + (x_{ij} \cdot \eta)^2\}^{1/2} \tag{3}$$

In the center-of-momentum frame, the transverse difference simplifies to the magnitude of the usual radial separation of the two particles, r_{ij}. There are three times in general, but in the center-of-momentum frame, the Hamiltonian depends on one time only, the time of the center of momentum or the rest frame of the proton.

The "lab frame" four-momenta of the particles are p_1, p_2, and p_3 and the masses are m_1, m_2, and m_3. We introduce the center-of-mass and relative four-momenta by

$$\begin{aligned} \mathbf{P} &= p_1 + p_2 + p_3 \\ \pi_1 &= (m_2 p_1 - m_1 p_2)/m \\ \pi_2 &= [m p_3 - m_3 (p_1 + p_2)]/M \end{aligned} \tag{4}$$

where for convenience we introduced

$$m = m_1 + m_2 \qquad \text{and} \qquad M = m_1 + m_2 + m_3$$

π_1 and π_2 are four-dimensional generalizations of the Jacobi vectors commonly used in three-body studies. We also define the center-of-mass coordinates and the relative four-vector coordinates through the equations

$$\begin{aligned} MR &= m_1 x_1 + m_2 x_2 + m_3 x_3 \\ r_1 &= x_1 - x_2 \\ m r_2 &= -m_1 x_1 - m_2 x_2 + m x_3 \end{aligned} \tag{5}$$

These equations can be inverted, of course, resulting in

$$\begin{aligned} x_1 &= R + m_2 r_1/m - m_3 r_2/M, & p_1 &= m_1 P/M + \pi_1 - m_1 \pi_2/m \\ x_2 &= R - m_1 r_1/m - m_3 r_2/M, & p_2 &= m_2 P/M - m_2 \pi_2/m - \pi_1 \\ x_3 &= R + m r_2/M, & p_3 &= m_3 P/M + \pi_2 \end{aligned} \tag{6}$$

If we substitute the relative momenta for the individual momenta, the zeroth or time coefficients of π_1^0 and π_2^0 vanish. The only time component surviving in the equation is the center-of-mass time conjugate to P^0, as we obtain

$$\left\{\begin{array}{c} \gamma^0\gamma^0\gamma^0 P^0 + (-\vec{\gamma} \cdot \vec{P}\, m_1/M - m_1)\gamma^0\gamma^0 \\ + \gamma^0\,(-\vec{\gamma} \cdot \vec{P} m_2/M - m_2)\,\gamma^0 + \gamma^0\gamma^0(-\vec{\gamma} \cdot \vec{P} m_3/M - m_3) \\ - \vec{\gamma} \cdot \vec{\pi}_1\gamma^0\gamma^0 + \gamma^0\vec{\gamma} \cdot \vec{\pi}_1\gamma^0 + \vec{\gamma} \cdot \vec{\pi}_2\gamma^0\gamma^0 m_1/m \\ + \gamma^0\vec{\gamma} \cdot \vec{\pi}_2\gamma_0 m_2/m - \gamma^0\gamma^0\vec{\gamma} \cdot \vec{\pi}_2 - (+V_{12} + V_{23} + V_{31}) \end{array}\right\} \times \Phi = 0 \tag{7}$$

The ordering in each term specifies the particle on which the gamma matrices apply. Multiplying the above by a γ^0 for each particle allows one to solve for P^0, the center-of-mass energy, and the Hamiltonian equation results. We solve in the center-of-momentum frame, where the total three-momentum vector $\vec{P}$ is zero.

The QCD equations for the potentials from three quarks in a color singlet, when solved, probably will result in color-dependent potentials. The potentials indicated here are presumably then the result from averages over such color dependence. In the nonrelativistic approach, the Lorentz characteristics of the potential need not be specified. For the relativistic case, a central diagonal spin-independent potential has previously been used (Strobel and Shitikova, 1996; Strobel and Shitikova, 1997; Semay *et al.*, 1993). The confining potential considered here is

$$V_{12} = [7 + 4\beta_1 + 4\beta_2 + \beta_1\beta_2]\, cr_{12}/16 \tag{8}$$

where c is a constant adjusted to reproduce the proton rest energy. The coefficients have been normalized to sum to unity, be symmetric with respect to particle exchange, as they must for identical quarks, and to satisfy the Semay condition (Semay *et al.*, 1993) for avoiding the Klein paradox. The linear confinment dependence is a way of treating strongly interacting QCD fields as lying along flux tubes between the quarks. The detection of only particles that are color singlets can be interpreted as indicating that color field lines all start or end on quarks or antiquarks. The linear confinment potential comes from assuming that the field energy lies along flux tubes of constant cross-sectional area, such that the quark potential energy is then proportional to the distance between the interacting quarks. The Hamiltonian is treated as independent of color.

The composite three-quark wave function Φ is written as

$$\Phi = \psi_{\text{color}}\psi_{\text{fc}}\psi_{\text{space}} \tag{9}$$

The color singlet part consists of a color factor,

$$\psi_{\text{color}} = \det(abc)/\sqrt{6} \tag{10}$$

where a, b, and c denote the three color indices of the quarks. This color factor is totally antisymmetric upon exchange of color indices of the three quarks. There is also a totally symmetric flavor and angular momentum coupling part of the composite three-quark wave function ψ_{fc} which can be expressed as the sum of two parts:

$$\psi_{\text{fc}} = (\chi_s[j_1, j_2]1, j_3 JM_z\rangle + \chi_A[j_1, j_2]0, j_3 JM_z\rangle)/\sqrt{2} \tag{11}$$

χ_s and χ_a are the flavor parts of the wave function that are symmetric and antisymmetric, respectively, upon exchange of the first pair of coordinates. For the proton, J is one-half. The orbital part of the wave function for each quark is a two-component Dirac spinor coupled, for example, for particle 1 as

$$\phi^m_{j\pi}(r_1) = \begin{bmatrix} C_l(r_1/\rho)^l Y^{ml}_l \zeta_{1/2}[l,\ 1/2]jm\rangle \\ i\bar{\sigma}_1 \cdot \hat{r}_1 C_{l'}(r_1/\rho)^{l'} Y^{ml'}_{l'} \zeta_{1/2}[l',\ 1/2]jm\rangle \end{bmatrix} \tag{12}$$

l and l' are determined from the total angular momentum j and the parity π of the orbital the quark occupies. There is also an eight-component hyperradial dependent part of the composite three-quark wave function. After the hyperangular integration is done, the Hamiltonian is a set of eight coupled first-order linear differential equations involving the eight hyperradial components as unknowns to be solved for. The details are the same as in Strobel (1996b). The hyperspherical coordinates are used. These consist of a hyperradius and five hyperangles. One possible set of these coordinates is as follows. The location of the three masses determines a triangle locating the masses at the corners. The triangle has a normal. The spherical polar coordinates of the normal are the first two hyperangles. Any two interior angles of the triangle are the next two hyperangles. The azimuthal orientation of the triangle about the normal is the fifth hyperangle. The hyperradius is given as

$$\rho^2 = r_1^2 + r_2^2 + r_3^2 = 2r^2/3 \tag{13}$$

2.1. The Hyperangular Separation

The composite three-quark wave function is expanded into hyperspherical harmonics and the integration over hyperangles is done (Baz and Zhukov, 1970). The result is analogous to the two-body expansion in spherical harmonics. For a two-body problem with a potential V, the Schrödinger equation is

$$[-\hbar^2\nabla^2/2m + V(\bar{r})]\Psi = E\Psi \tag{14}$$

Expanding in spherical harmonics and integrating over angles results in

$$
\begin{aligned}
&[-(\hbar^2/2m)\, d^2/dr^2 + L(L+1)/2mr^2 - E]R_L Y_{LM} \\
&\quad = -\sum \langle LM|V|L'\ M'\rangle R_{L'}\ Y_{L'M'}
\end{aligned} \tag{15}
$$

Now if the potential has a spherical symmetry in the two-body case, the right-hand side reduces to a single term where $L'M'$ equals LM, and only a single term of the summation survives and the equations uncouple. In the three-body case, if the potential is a quadratic function of the separation between the interacting particles, then the sum of the potentials may depend only on the hyperradius, and similiarly the summation over hyperharmonics reduces to a single term. A linear confining potential with an attractive Coulombic term is used here. After the hyperangular integration, the equations are solved in hypercentral approximation, namely only the term diagonal in quantum numbers is retained of the potential matrix element, and the nondiagonal terms are neglected, but are not zero. The equations that result are a set of eight coupled first-order differential equations in one variable, the hyperradius. The unknowns are the eight hyperradial dependent components of the space part of the three-quark composite wave function, R_i.

With equal *u, d* quark masses, all quarks in the proton have the same mass. For the $(1/2^+)^3$ configuration considered, each quark has the same set of quantum numbers. Hence the eight-component composite wave function has the symmetry that $R_2 = R_3 = R_5$ and that $R_4 = R_6 = R_7$. The eight-component Hamiltonian equation, after hyperangular integration, simplifies to a set of coupled linear equations involving the four unknown hyperradial components. For the $(1/2^+)^3$ configuration, these equations are given by

$$
\begin{bmatrix}
(3M - E + V_1) & -D(5) & 0 & 0 \\
D(0) & (M - E + V_2) & -D(6)/2 & 0 \\
0 & 2D(-1) & (-M - E + V_4) & -D(7)/5 \\
0 & 0 & 3\mathrm{D}(-2) & (-3M - E + V_8)
\end{bmatrix}
\times
\begin{bmatrix}
R_1 \\ R_2 \\ R_4 \\ R_8
\end{bmatrix}
= 0 \tag{16}
$$

This matrix operates on the hyperradial components R_1, R_2, R_4, and R_8. The relativistic kinetic energy appears in the operator

$$
D(n) = d/dr + n/r \tag{17}
$$

After integrating over the hyperangles, the norm for the composite three-quark wave function in terms of the eight components R_i is

$$1 = N^2\,(2/\pi^{3/2}) \sum_i \int \rho^5\, d\rho [R_i]^2 / \Gamma[(\Lambda + 6)/2] \tag{18}$$

$\Gamma(n)$ is the gamma function of order n, and Λ is twice the orbital angular momentum associated with a given component of the composite three-quark wave function, ranging from 0 to 3 for the configurations considered here.

The linear confining hypercentral potential and the solution determined from it are now presented. After the hyperangular integration, the sum of the three pairwise quark potentials is the hypercentral potential:

$$V_i = a_i r + k_i + b_i/r \tag{19}$$

The solution and the potential constants are expressed in terms of the proton rest energy E, the quark mass M, and a size parameter L determined by reproducing the rms charge radius of the proton:

$$\begin{array}{ll} \text{component} & \text{component constant} \\ R_1 = A\exp(-Lr) & A = 1 \\ R_2 = R_3 = R_5 = Br\exp(-Lr) & B = -(E - 3M)/6 \\ R_4 = R_6 = R_7 = Cr^2\exp(-Lr) & C = -(E - M)B/4 \\ R_8 = Dr^3\exp(-Lr) & D = -(E + M)C/2 \end{array} \tag{20}$$

The hypercentral potential constants are as follows:

index	Λ	a_i	k_i	b_i
1	0	$(E - 3M)L/6$	0	0
2	2	$(E - M)L/8$	0	$-6L/(E - 3M)$
4	4	$(E + M)L/10$	0	$-8L/(E - M)$
8	6	0	$(E + 3M)$	$-6L/(E + M)$

Solutions for other configurations can be found in Strobel (1996b). The linear confining hypercentral potential with a Coulombic attractive term leads to an exponential hyperradial dependence for each component of the composite three-quark wave function.

2.2. One-Body Operators

With F/r and G/r denoting, respectively, the upper and lower radial components of a particle obeying the one-body Dirac equation, the radial Dirac equation becomes

$$[M + S - E + V]F + [(k/r) - d/dr]G = 0 \tag{21}$$

$$[(k/r) + d/dr]F + [-M - S - E + V]G = 0 \tag{22}$$

Here, as usual, k is plus or minus $(J + 1/2)$, depending on the total angular momentum J and the parity. V and S are the zeroth components of a vector potential and a scalar potential respectively. The normalization is

$$1 = \int [F^2 + G^2]\, dr \tag{23}$$

The axial charge is

$$G_a = (5/3) \int [F^2 - G^2/3]\, dr \tag{24}$$

The magnetic moment operator is

$$\mu = I \cdot \text{area} = e\bar{r} \otimes \bar{v}/2 = ec\bar{r} \otimes \bar{\alpha}/2 \tag{25}$$

Here $\bar{\alpha}$ are the Dirac equation matrices. For a particle of electric charge e confined into a bound state characterized by the energy E and by k, the magnetic moment (Perelomov and Popov 1970; Smith and Lewin, 1980) is

$$\mu = [4kEe/(4k^2 - 1)] \int [FrG]\, dr \tag{26}$$

2.3. Failure of Schmidt-Limit Conditions for Quarks

The Schmidt limit is successful for predicting the magnetic moment for odd-A nuclei. It comes from assuming that the magnetic moment is due solely to the last unpaired nucleon. The nucleon's motion is assumed to be described by the Dirac equation, and the limit is taken that the energy E approaches the mass M of the bound fermion. The asymptotic potentials V and S must be negligible compared to the mass (Miller, 1975; Marganeau, 1940). For the $k = -1$, $(1/2^+)$ state, a consequences is that the magnetic moment in nuclear magnetons is $\mu = e\hbar/2mc$. This limit does not apply to bound quarks with e and m just becoming the quark charge and mass if the quarks are bound by confining potentials. We consider quadratic and linear confining potentials here for the quarks. This means the conditions for the Schmidt limit to apply are not met for bound quarks, as the asymptotic potentials are not small compared to the quark mass or energy. For the one-body Dirac equation $(1/2^+)$ state and a quadratic confining potential the equation can be solved for F and G analytically, resulting in the magnetic moment becoming, in nuclear magnetons,

$$\mu = 4M_p/(3E - M) \tag{27}$$

Even if the mass M is set to the proton mass M_p, as the energy E approaches the mass, the magnetic moment is 2 NM, not 1 NM for this quadratic confining potential.

With a linear confining potential including a constant and an attractive Coulombic potential term, the solution to the $k = -1$ Dirac equation is

$$F = A \exp(-Lr) \quad \text{and} \quad G = Br \exp(-Lr) \qquad \text{with} \quad B/A = -(E - M)/3$$

The magnetic moment for the linear confining potential is then found to be

$$\mu = 4(E - M)M_p/[3L^2 + (E - M)^2] \tag{28}$$

The confining potential is proportional to the size parameter. In the nonrelativistic limit where E approaches M, the Dirac magnetic moment vanishes in the linear confining potential case. This difference illustrates one weakness of the constituent quark approach to the magnetic moment of confined fermions (Strobel, 1997). The Schmidt value for the magnetic moment does not tie directly to the quark mass with potentials that are not asymptotically small in comparison to the quark mass or energy. This difference carries over to the three-body Dirac equation results for magnetic moments. Small current quark masses are used here. The magnetic moments are determined from their composite three-quark wave function expectation values and not from the Schmidt limits.

3. CALCULATIONS OF REST FRAME PROTON PROPERTIES

3.1. Nodes in the Wave Function

The positive-parity $(1/2^+)^3$ configuration and the negative-parity $(1/2^-)^3$ configurations are quite similiar in the massless limit where the large and small components just replace each other up to a possible phase. In the nonrelativistic limit with large quark masses of one-third of the proton mass, these configurations have 0 and 3 nodes, respectively. However, in the extreme relativistic small-quark-mass limit of interest here, the number of nodes is about the same for either configuration. For the linear confining hypercentral potential, when the potential length parameter is fixed at $L = 0.538$ GeV, to reproduce the proton rms charge radius, the number of nodes is 1.30. This is the expectation value of the number of nodes in the wave function associated with each component of the eight-component composite wave function. The number of nodes versus assumed quark mass can be seen for both configurations in Fig. 1. The number of nodes decreases to zero for the positive-parity configuration as the quark mass approaches one-third of the proton mass. For the negative-parity configuration, the number of nodes rises toward 1.65 in the same limit. In the nearly massless limit, both configurations predict a similiar value for the number of nodes.

3.2. Magnetic Moment Predictions

The linear confinement potential parameter L that results in reproducing the proton rms charge radius is shown in Fig. 2. For small quark masses of a few MeV, the length parameter is about 0.538 GeV. For larger quark masses the length parameter decreases to about 0.36 GeV. In Fig. 3 the Dirac magnetic moment is shown for massless quarks, and for 50-MeV quark masses. The higher curve, for massless quarks, applies to either a $(1/2^+)^3$ or a $(1/2^-)^3$ configuration. The $(1/2^+)^3$ configuration predicts a monotonically lower Dirac magnetic moment as the quark mass is increases. The $(1/2^-)^3$ configuration predicts an increasing Dirac magnetic moment as the quark mass increases, holding the potential length parameter constant. For a size parameter that reproduces the proton rms charge radius, the Dirac magnetic moment is about 2.18 NM for the $(1/2^+)^3$ configuration. The mixed configurations of $(1/2^-)^2(1/2^+)$ or $(1/2^+)^2(1/2^-)$ result in Dirac magnetic moments systematically about

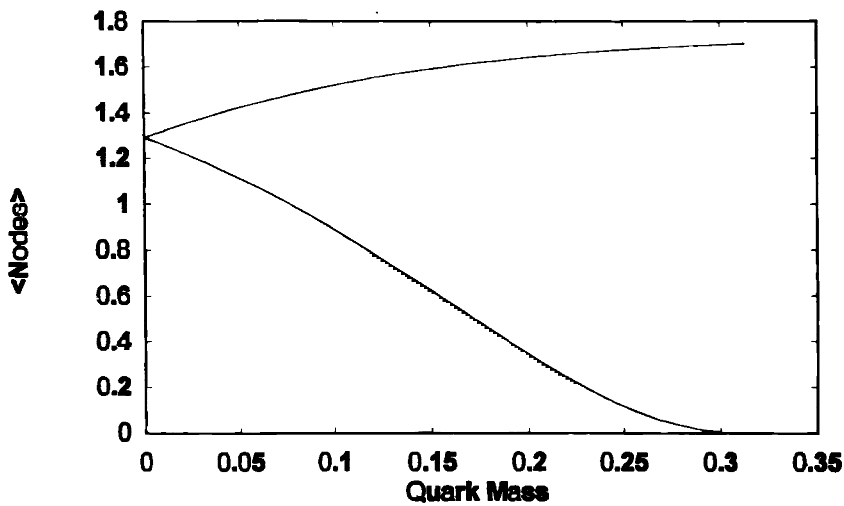

Fig. 1. Node expectation value for the $(1/2^+)^3$ and the $(1/2^-)^3$ configurations versus the quark mass in GeV. The length parameter of the linear confining hypercentral potential is fixed at $L = 0.538$ GeV to reproduce the proton rms charge radius. The upper curve is for the negative-parity configuration, the lower curve is for the positive-parity configuration.

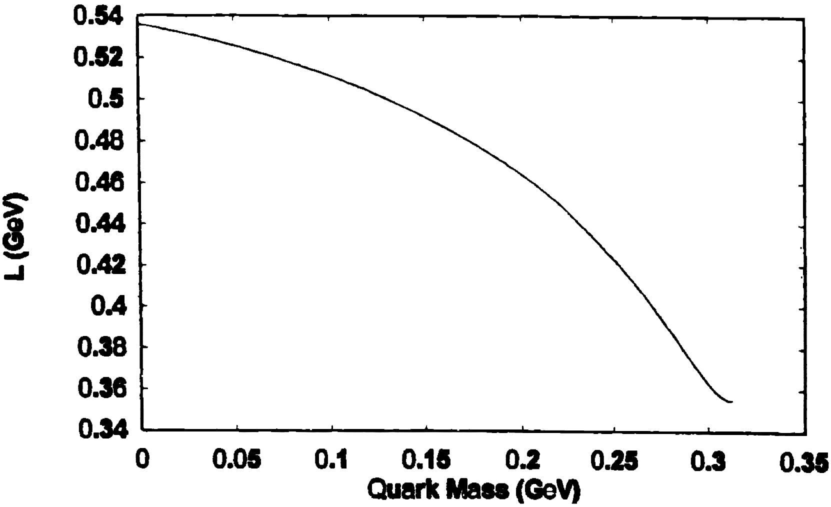

Fig. 2. The length parameter of the linear confinement hypercentral potential that reproduces the proton rms charge radius versus quark mass, for the $(1/2^+)^3$ configuration.

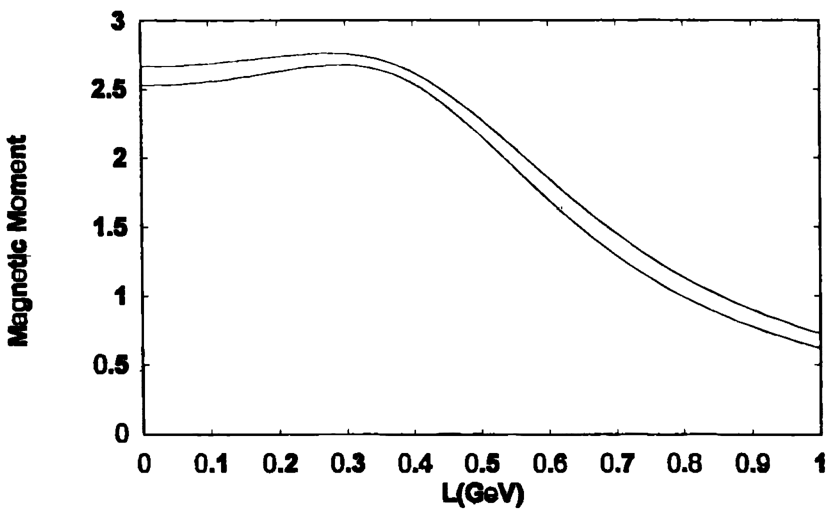

Fig. 3. Dirac magnetic moment for the proton with the linear confining hypercentral potential. The upper curve is for both the $(1/2^+)^3$ and $(1/2^-)^3$ configurations of positive and negative parity, respectively, with massless quarks. The lower curve is the $(1/2^+)^3$ configuration of positive parity with 50-MeV quark masses.

one-third of the symmetric $(1/2^+)^3$ configuration for the same parameters. For this reason these mixed configurations are dropped from further consideration as having major contributions to the proton ground-state wave function.

Quarks are assumed to have an anomalous magnetic moment that contributes to the proton magnetic moment in addition to the Dirac magnetic moment. The anomalous magnetic moment of a bound fermion depends on all components of the wave function. The anomalous magnetic moment of a bound quark is calculated following Marganeau (1940) and Miller (1975).

For equal masses and for the $(1/2^+)^3$ configuration the anomalous moment from the quarks in NM is

$$\mu_a = \mu_0(2N^2/\pi^{3/2}) \int \rho^5 \, d\rho[(R_1^2/2) + (7R_2^2/18) + (5R_4^2/72) + (R_8^2/360)] \quad (29)$$

and μ_0 is the anomalous magnetic moment for a "free valence quark" of $(\alpha/2\pi)e/m$, where α is now the fine structure constant, 1/137 .... Keeping the linear potential parameter constrained to reproduce the proton rms charge radius, the predicted magnetic moment, Dirac plus anomalous, for the $(1/2^-)^3$ configuration is shown in Fig. 4. The predicted moment peaks at about 2 NM for this configuration. There may also be pionic, strange, or sea quark contributions to the proton magnetic moment. These are neglected here in the three-quark model of the proton. For the $(1/2^+)^3$ configuration, using

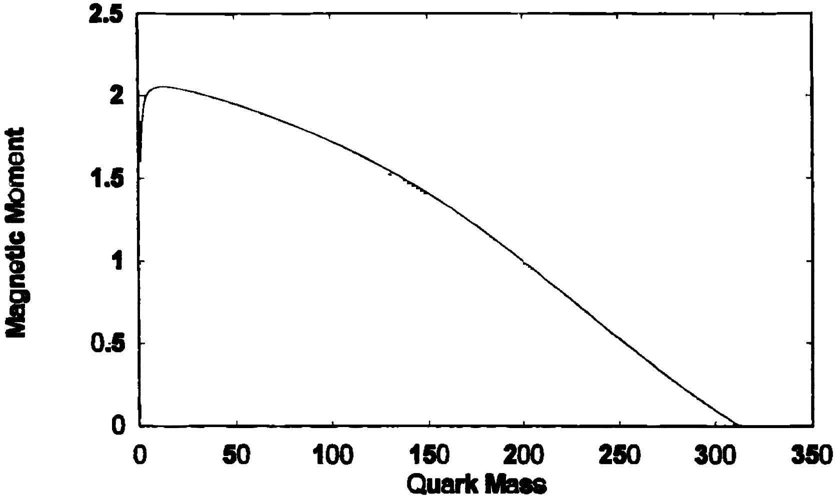

Fig. 4. The Dirac plus anomalous magnetic moment for the proton from the $(1/2^-)^3$ configuration with the length parameter constrained to reproduce the proton rms charge radius.

the linear confining potential, a quark mass can be detemined such that the Dirac plus anomalous magnetic moment reproduces the experimental value of 2.793 NM. This assumes the u and d quark masses are the same. The quark masses determined in this way are all less than 9 MeV, as can be seen in Fig. 6. For the potential parameter that reproduces the proton rms charge radius, this suggests quark masses of about 1 MeV. This has neglected any nonvalence quark contribution to the proton magnetic moment, such as pionic, sea, or strange quark. To the extent that their inclusion increases the proton magnetic moment, the required quark anomalous magnetic moment would decrease. The quark mass determined this way would increase. Therefore these masses are only a lower limit to the quark masses within the proton.

For equal quark masses, there is no anomalous quark magnetic moment contribution for the neutron. The neutron magnetic moment is calculated as a function of assumed quark mass, keeping the rms proton charge radius as a constraint on the potential length parameter L. The $(1/2^{+})^{3}$ configuration result can be seen in Fig. 5, where the neutron proton mass difference has been included in the composite three-quark wave function. The neutron magnetic moment is reproduced only for small quark masses. As the quark mass approaches one-third of the neutron rest energy, the Dirac magnetic moment goes to zero. This is understandable, as this behavoir is also seen in the earlier simplified one-body Dirac equation analysis of a linear confining potential.

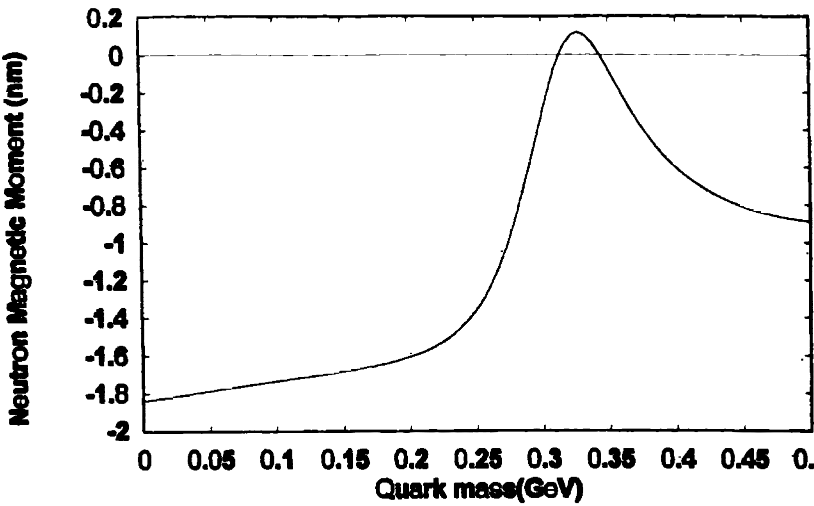

Fig. 5. The Dirac magnetic moment of the neutron for the $(1/2^{+})^{3}$ configuration with the length parameter constrained to reproduce the proton rms charge radius. Small quark masses result in best agreement with experiment for the magnetic moment.

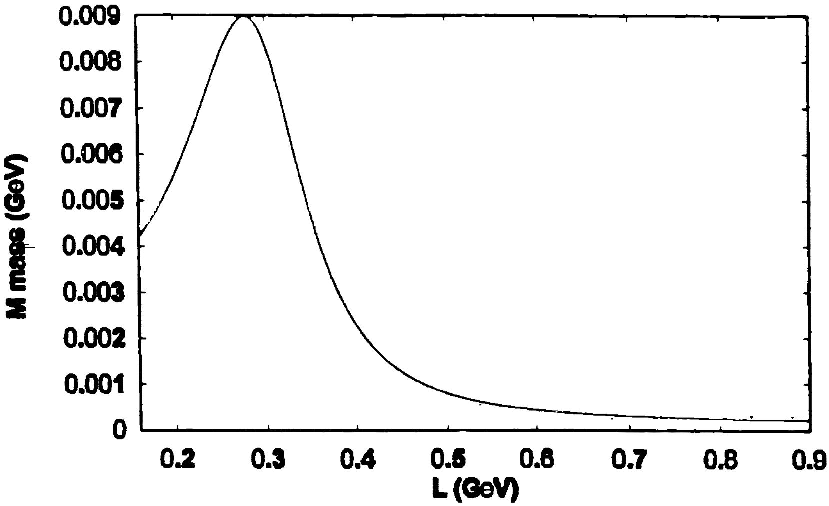

Fig. 6. Average quark masses determined by requiring the Dirac plus anomalous magnetic moment of the $(1/2^{+})^{3}$ configuration of the valence quarks to reproduce the proton magnetic moment.

3.3. Proton Axial Charge

We now consider the axial charge of the proton. This is primarily related to the difference in the upper versus lower component contributions to the composite wave function normalization. The linear confining hypercentral potential model predicts axial charges close to the experimental value of 1.26. Figure 7 shows the predictions versus the linear potential length parameter. The middle curve is for massless quarks, with the same results for the $(1/2^{-})^{3}$ configuration as for the $(1/2^{+})^{3}$ configuration. When the potential length parameter reproduces the proton charge rms radius, the axial charge calculated is 1.20 for either configuration. When the quark mass is 10 MeV, the $(1/2^{+})^{3}$ configuration reproduces the proton axial charge as well as the proton rms charge radius.

The axial charge predicted by the $(1/2^{-})^{3}$ configuration decreases with assumed quark mass, keeping the linear confining potential length parameter fixed. This can be seen in Fig. 8. This configuration is not able to simultaneously reproduce the proton magnetic moment and axial charge when reproducing the rms charge radius in this model.

3.4 Quark Helicity in the Proton

HERMES data analyses confirm that quarks carry about 25–30% of the neutron spin (Watson, 1997). Abe *et al.* (1995) have measured the quark

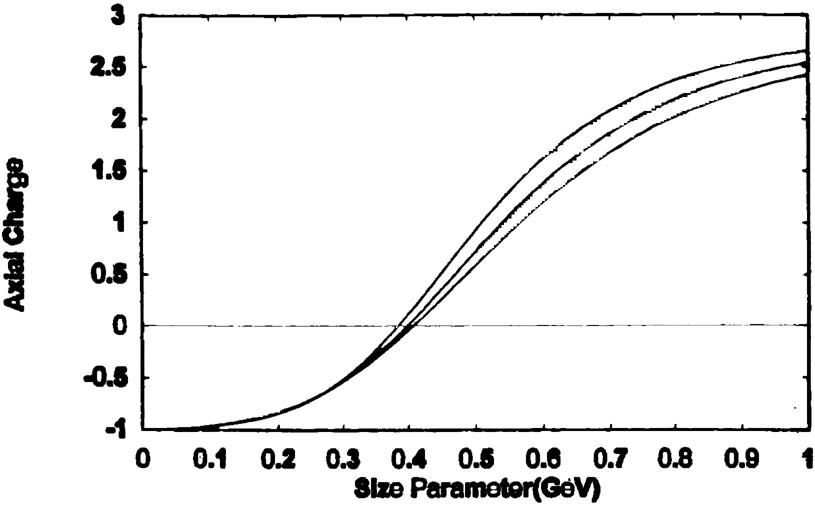

Fig. 7. Proton axial charge versus length parameter of the linear confinement hypercentral potential. Middle curve is for both the $(1/2^{+})^{3}$ and the $(1/2^{-})^{3}$ configurations with massless quarks. Upper curve is for 10-MeV quark masses for the $(1/2^{+})^{3}$ configuration. Lower curve is for 10-MeV quark masses for the $(1/2^{-})^{3}$ configuration.

contribution to the helicity of the proton as 0.27 ± 0.10. Using the linear confining potential with the length parameter L chosen to reproduce the proton rms charge radius, the quark contribution to the proton helicity is 0.253, in good agreement with experiment. It is noted that only for the component that survives in the nonrelativistic limit is there no orbital angular momentum contribution to the nucleon spin. The upper component of the quark wave function contribution to the norm can be seen in Fig. 9 for both the $(1/2^{+})^{3}$ and the $(1/2^{-})^{3}$ configurations. The potential length parameter is constrained to reproduce the proton rms charge radius. For massless quarks these are 0.585 and 0.415, respectively. The upper component contribution does not dominate the norm. The lower component contribution is always comparable to the upper. This shows that including all components of the composite wave function in the three-body Dirac equation is necessary for these potentials and small quark masses.

4. SUMMARY

The proton is modeled as three quarks of small current quark mass. The three-body Dirac equation is solved with a spin-independent central diagonal linear confining potential with an attractive Coulombic term, in hypercentral approximation. Several configurations are considered, the $(1/2^{-})^{3}$, $(1/2^{+})^{3}$,

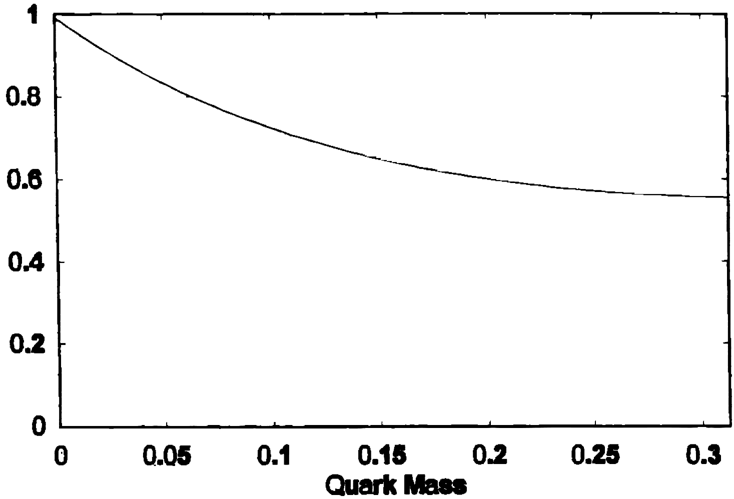

Fig. 8. The axial charge versus assumed quark mass for the $(1/2^-)^3$ configuration.

$(1/2^+)^2(1/2^-)$, and $(1/2^+)^2(1/2^-)$ configurations. The latter mixed configurations produce Dirac magnetic moments that are too small. These configurations are deemed as not important contributors to the proton ground-state wave function. For the symmetric configurations, the $(1/2^+)^3$ configuration better reproduces the proton rest frame properties than does the $(1/2^-)^3$ configuration. The linear confining potential shape used can be inferred from

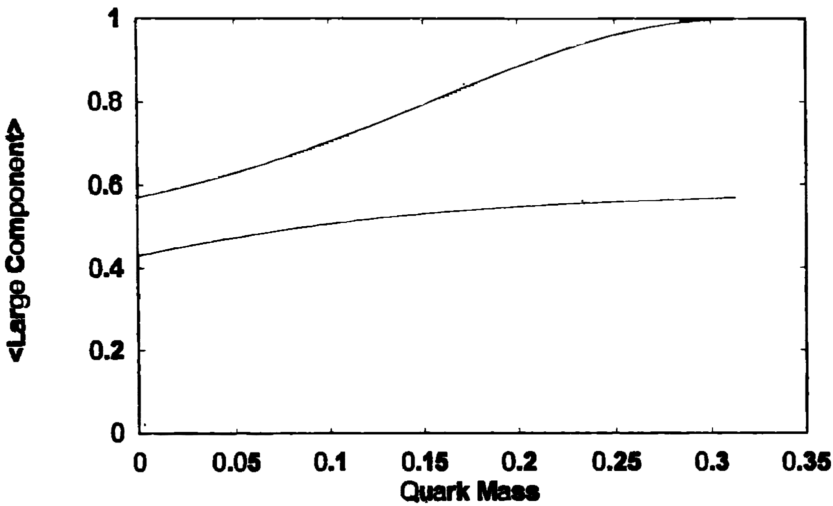

Fig. 9. Upper component contribution to the norm, upper curve $(1/2^+)^3$ configuration, lower curve $(1/2^-)^3$ configuration.

QCD theory. The potential avoids the Klein paradox. Conditions for the Schmidt limits for the magnetic moments of bound fermions are not met for quarks with confining potentials that are not asymptotically small. The quarks are assumed to possess anomalous magnetic moments like those of the electron and muon. Assuming all three quarks have equal mass allows fits to the axial charge and magnetic moment to discriminate in favor of the $(1/2^+)^3$ configuration in this model. This relativistic three-quark model with small quark masses is able to reproduce the charge rms radius, magnetic moment, axial charge, and energy of the proton, using a $(1/2^+)^3$ configuration of quarks. The proton rest energy is mostly potential energy and quark kinetic energy, with little coming from the rest masses of the three quarks.

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